A Compositional and Monotone Approach to Termination

Towards a more dependable and scalable termination analysis

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Background

program correctness = partial correctness + termination proof

It does the right thing if it terminates

It does terminate

Programs as transition systems

- A program statement defines a transition on the state space S
- Transition system $\langle S, R \rangle$
  - S: state space
  - $R \subseteq S \times S$: transition relation

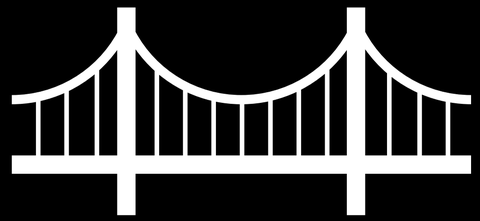
Ranking functions

- Loop terminates iff it has a ranking function
- Ranking function r for a loop with state space S and body R
  - $r : S \to B$ is a mapping from state space S into a set B equipped with a well-founded relation $\succ$
  - $(s_1, s_2) \in R \Rightarrow r(s_1) \succ r(s_2)$
- Examples: linear ranking functions, lexicographic linear ranking functions

```
// ranking function: x
while (x > 0) {
    x = x - 1;
}
```

```
// ranking function: < i, j >
while (i > 0) {
    if (j > 0) {
        j--;
    } else {
        j = 100;
        i--;
    }
}
```

Theory



Practice

- Complete procedure for lexicographic ranking function synthesis for a single loop (Gonnord et al., 2015)
- Deciding termination of linear transition systems (Tiwari, 2004)
- Weakest terminating pre-conditions for octagonal relations (Bozga et al., 2012)
- …

- Repeatedly sample a lasso-shaped trace and synthesize ranking function for it (Cook et al., 2006)
- Extend single-loop ranking function synthesis to whole program termination by analyzing paths between cut points (Gonnord et al., 2015)
- Guess ranking function candidates, and then synthesize conditions under which they become real ranking functions (Cook et al., 2008)
- …

Motivation

“Because Infer is *compositional*, it can operate incrementally, quickly producing results on a code diff even if the diff is part of a codebase in the millions of lines”

— **Peter O’Hearn**

Want: a compositional analysis of termination

analyze each loop in isolation and combine the analysis results

It brings us: parallelism and scalability, incremental analysis, etc.

Want: a compositional analysis of termination

Requires: an analysis that generates sufficient conditions for termination

- Consider a compositional analysis for termination that looks at each loop in isolation
- When analyzing the while loop, we don’t know what values could x take
- The analysis need to figure out a sufficient condition for termination and checks if the context implies that condition

```
if (__VERIFIER_nondet_int() ≠ 0) {
    x = 1;
} else {
    x = -1;
}

// conditions: x > 0 or x < 0
while (y < 100 && z < 100) {
    y = y + x;
    z = z - x;
}
```

“Tools can understand our programs, but we cannot understand our tools”

— Leino and Moskal

Heuristics

widely used in modern tools, but lead to unpredictability in the analysis

Ultimate Automizer

- #1 tool in Software Verification Competition termination category
- Supports linear, lexicographic linear, multiphase, piecewise ranking functions, etc.
- Can prove this program terminates within a few seconds, but …

```
int main() {
    int x, y, z, a, b;
    int n;

      while (z ≥ 0 && __VERIFIER_nondet_int()) {
          z = z - 1;
      }

      if (a == b) {
          while (x ≥ 0 && y ≥ 0) {
              y = y - z;
              while (z > 0) {
                  z = z - 1;
                  y = y + 2 * z - x;
              }
              x = x + a - b - 1;
          }
      }

    return 0;
}
```

Ultimate Automizer

- #1 tool in Software Verification Competition termination category
- Supports linear, lexicographic linear, multiphase, piecewise ranking functions, etc.
- Can prove this program terminates within a few seconds, but it will not terminate after 30 minutes on this slightly modified program which ought to be “easier” to reason about

```
int main() {
    int x, y, z, a, b;
    int n;

    if (z > 999999) {

        while (z ≥ 0 && __VERIFIER_nondet_int()) {
            z = z - 1;
        }

        if (a == b) {
            while (x ≥ 0 && y ≥ 0) {
                y = y - z;
                while (z > 0) {
                    z = z - 1;
                    y = y + 2 * z - x;
                }
                x = x + a - b - 1;
            }
        }

    }

    return 0;
}
```

Want: a predictable analysis of termination

produces a more precise result when given more precise information

Theory

Practice

This work presents a termination analysis that is:

- **Predictable**
  - Adding information to program leads to more precise analysis results
- **Compositional**
  - Analyzing on a per-loop basis

A monotone and compositional analysis for conditional termination

How we achieve compositionality and predictability?

- **Compositionality: compute termination conditions for a program by analyzing its components and composing the results**
  - Extend Tarjan's framework to termination analysis
- **Predictability: give a monotone procedure that computes a sufficient terminating precondition for a program**
  - Exploit the compositional framework
    - Reduce termination of programs to termination of single loops
    - Access to a summary for the body of each loop
  - Utilize decision procedure for linear loops to generate terminating pre-conditions for summaries
    - Abstract transition formulas into linear models that can be handled by Tiwari's method

An algebraic framework for program analysis

- (Tarjan, 1981) views the control flow graph of the program as a labelled graph where labels are transitions relations
  - Can efficiently compute regular expressions that recognize all paths between two program locations
  - Certain program analysis can be defined using interpretation of regular expressions that represent sets of paths

$$a \in \Sigma$$

$$e \in \text{RegExp}(\Sigma) ::= a \mid 0 \mid 1 \mid e_1 + e_2 \mid e_1 e_2 \mid e^*$$

Interpreting path expressions as transition formulas

- Transition formula $\mathbf{TF}$
  - A formula in linear integer arithmetic over program variables and primed program variables, and loop counter
  - Describes a transition relation
- Compositional recurrence analysis (Kincaid et al., 2015) interprets path expressions as transition formulas

$$x \in \mathsf{Var}$$
$$n \in \mathbb{Z}$$
$$y \in \mathsf{BoundVar}$$
$$t \in \mathsf{Term} ::= x \mid x' \mid y \mid n \mid n \cdot t \mid t_1 + t_2$$
$$T \in \mathsf{TF} ::= t_1 < t_2 \mid t_1 = t_2 \mid T_1 \wedge T_2 \mid T_1 \vee T_2 \mid \exists y \,.\, T$$

$$T_1 +^{\mathcal{T}} T_2 \triangleq T_1 \vee T_2$$
$$0^{\mathcal{T}} \triangleq \mathrm{False}$$
$$1^{\mathcal{T}} \triangleq \bigwedge_{x \in \mathsf{Var}} x' = x$$
$$T_1 \cdot^{\mathcal{T}} T_2 \triangleq \exists \mathsf{Var}'' \,.\, T_1[\mathsf{Var}' \mapsto \mathsf{Var}''] \wedge T_2[\mathsf{Var} \mapsto \mathsf{Var}'']$$
$$T^{*^{\mathcal{T}}} \triangleq \ldots$$
$$\mathcal{T}[\![(u, x := e, v)]\!] \triangleq x' = e \wedge \bigwedge_{y \neq x \in \mathsf{Var}} y' = y$$
$$\mathcal{T}[\![(u, \mathsf{assume}(c), v)]\!] \triangleq c \wedge \bigwedge_{x \in \mathsf{Var}} x' = x$$

An algebraic framework for compositional termination analysis

We augment Tarjan's framework with ω-regular expressions:

- State formula $\mathbf{SF}$
  - formula in LIA over program variables that describes a set of program states
- Termination analysis defined as interpretation of ω-path expressions
  - atc: $\mathbf{TF} \to \mathbf{TF}$, approximate transitive closure operator provides the summary of loop body (Kincaid et al., 2015)
  - swf: $\mathbf{TF} \to \mathbf{SF}$, gives a set of initial states from which the transition is guaranteed to terminate
  - Let p be the set of ω-paths starting at program entry. If s if a state from which there is an non-terminating execution, then $s \models \mathcal{T}(p)$

$$a \in \Sigma$$
$$e \in \mathsf{RegExp}(\Sigma) ::= a \mid 0 \mid 1 \mid e_1 + e_2 \mid e_1 e_2 \mid e^*$$
$$f \in \omega\text{-}\mathsf{RegExp}(\Sigma) ::= e^\omega \mid e; f \mid f_1 \oplus f_2$$

$$S^{\mathcal{T}} \triangleq \mathbf{TF}$$
$$T_1 +^{\mathcal{T}} T_2 \triangleq T_1 \vee T_2$$
$$0^{\mathcal{T}} \triangleq \mathrm{False}$$
$$1^{\mathcal{T}} \triangleq \bigwedge_{x \in \mathsf{Var}} x' = x$$
$$T_1 \cdot^{\mathcal{T}} T_2 \triangleq \exists \mathsf{Var}'' . T_1[\mathsf{Var}' \mapsto \mathsf{Var}''] \wedge T_2[\mathsf{Var} \mapsto \mathsf{Var}'']$$
$$T^{*\mathcal{T}} \triangleq \mathsf{atc}(T)$$
$$\mathcal{T}[\![(u, x := e, v)]\!] \triangleq x' = e \wedge \bigwedge_{y \neq x \in \mathsf{Var}} y' = y$$
$$\mathcal{T}[\![(u, \mathsf{assume}(c), v)]\!] \triangleq c \wedge \bigwedge_{x \in \mathsf{Var}} x' = x$$
$$L^{\mathcal{T}} \triangleq \mathbf{SF}$$
$$T^{\omega^{\mathcal{T}}} \triangleq \neg \mathsf{swf}(T)$$
$$T;^{\mathcal{T}} S \triangleq \exists \mathsf{Var}' . T \wedge S[\mathsf{Var} \mapsto \mathsf{Var}']$$
$$S_1 \oplus^{\mathcal{T}} S_2 \triangleq S_1 \vee S_2$$

How we achieve compositionality and predictability?

- **Compositionality: compute termination conditions for a program by analyzing its components and composing the results**
  - Extend Tarjan's framework to termination analysis ✓
- **Predictability: give a monotone procedure that computes a sufficient terminating precondition for a program**
  - Exploit the compositional framework
    - Reduce termination of programs to termination of single loops (swf operator)
    - Access to a summary for the body of each loop (atc operator) ✓
  - Utilize decision procedure for linear loops to generate terminating pre-conditions for summaries
    - Abstract transition formulas into linear models that can be handled by Tiwari's method

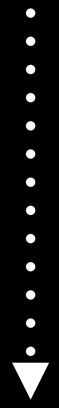
Termination analysis of the whole program

Define swf() operator that finds terminating pre-conditions for a transition formula

Compositional Recurrence analysis (Kincaid et al., 2015)

Reduce

in a monotone manner



Terminating preconditions for linear transition systems with rational eigenvalues based on (Tiwari, 2004)

Tiwari has a way to synthesize conditions for termination, but …

Long-term Dynamics of Linear Transition Systems

- Suppose the loop to the right does not terminate after k iterations, then what are the values of x, y, and z?

```
while (x > 0) {
    x = x + y;
    y = y + z;
}
```

- $$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
- $$\begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^k \begin{bmatrix} x^{(0)} \\ y^{(0)} \\ z^{(0)} \end{bmatrix}$$

Long-term Dynamics of Linear Transition Systems

- Suppose the loop to the right does not terminate after k iterations, then what are the values of x, y, and z?
  - $z^{(k)} = z^{(0)}$
  - $y^{(k)} = y^{(0)} + kz^{(0)}$
  - $x^{(k)} = x^{(0)} + ky^{(0)} + \frac{k(k-1)}{2} z^{(0)}$

```
while (x > 0) {
    x = x + y;
    y = y + z;
}
```

- Obviously the value of x affects termination, so we look at
  - $x^{(k)} = x^{(0)} + k(y^{(0)} - \frac{1}{2} z^{(0)}) + k^2 \frac{z^{(0)}}{2}$

Long-term Dynamics of Linear Transition Systems

- $x^{(k)} = x^{(0)} + k(y^{(0)} - \frac{1}{2}z^{(0)}) + k^2\frac{z^{(0)}}{2}$
- Consider when k gets large enough $k^2 \gg k \gg 1$
  - If $\frac{z^{(0)}}{2} \neq 0$ then $x^{(k)} \approx k^2\frac{z^{(0)}}{2}$
  - If $\frac{z^{(0)}}{2} = 0$ but $y^{(0)} - \frac{1}{2}z^{(0)} \neq 0$, then $x^{(k)} \approx k(y^{(0)} - \frac{1}{2}z^{(0)})$
  - If $\frac{z^{(0)}}{2} = 0, y^{(0)} - \frac{1}{2}z^{(0)} = 0$, then $x^{(k)} \approx x^{(0)}$

```
while (x > 0) {
    x = x + y;
    y = y + z;
}
```

Long-term Dynamics of Linear Transition Systems

If the loop never terminates, k *can* actually get large enough and $x^{(k)} > 0$ still holds

- If $\frac{z^{(0)}}{2} \neq 0$ then $x^{(k)} \approx k^2 \frac{z^{(0)}}{2} > 0 \Rightarrow \frac{z^{(0)}}{2} > 0$
- If $\frac{z^{(0)}}{2} = 0$ but $y^{(0)} - \frac{1}{2} z^{(0)} \neq 0$, then
  $x^{(k)} \approx k(y^{(0)} - \frac{1}{2} z^{(0)}) > 0 \Rightarrow y^{(0)} - \frac{1}{2} z^{(0)} > 0$
- If $\frac{z^{(0)}}{2} = 0$, $y^{(0)} - \frac{1}{2} z^{(0)} = 0$, then $x^{(k)} \approx x^{(0)} > 0$

```
while (x > 0) {
    x = x + y;
    y = y + z;
}
```

Long-term Dynamics of Linear Transition Systems

Thus non-termination implies the following:

- $\frac{z^{(0)}}{2} > 0$, or
- $\frac{z^{(0)}}{2} = 0 \ \wedge y^{(0)} - \frac{1}{2} z^{(0)} > 0$, or
- $\frac{z^{(0)}}{2} = 0 \wedge y^{(0)} - \frac{1}{2} z^{(0)} = 0 \wedge x^{(0)} > 0$

$$
\begin{array}{l}
z < 0 \\
\vee\, (z = 0 \wedge y < 0) \\
\vee\, (z = 0 \wedge y = 0 \wedge x \leq 0)
\end{array}
$$

```
while (x > 0) {
    x = x + y;
    y = y + z;
}
```

The negation of the above condition implies termination! (Tiwari, 2004)

Dominant Term Analysis (DTA) exploits long-time dynamics

- Using Tiwari's analysis to generate terminating conditions requires
  1. Finding loop guards expressed in linear terms of program variables (polyhedral guards)
  2. Linear loop update as matrix multiplication $\mathbf{x}' = A\mathbf{x}$
     - Exists a linear ordering on the eigenvalues of A

```
while  x > 0  {
    x = x + y;
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```

$$\begin{bmatrix} x^{(k)} \\ y^{(k)} \\ z^{(k)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^k \begin{bmatrix} x^{(0)} \\ y^{(0)} \\ z^{(0)} \end{bmatrix}$$

$$k^2 \gg k \gg 1$$

Dominant Term Analysis (DTA) exploits long-time dynamics

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Q: When is this possible? What if we have to compare $e^{\frac{k\pi}{3}i}$ and $e^{\frac{3k\pi}{4}i}$?
A: That is hard to do. Want to consider only matrices with real/rational eigenvalues

Tiwari has a way to synthesize conditions for termination, but …

Tiwari has a way to synthesize conditions for termination, but

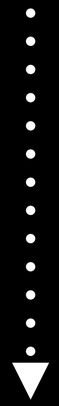
only applies to *linear transition systems with polyhedral guards and rational spectra*

Termination analysis of the whole program

Define swf() operator that finds terminating pre-conditions for a transition formula

Compositional Recurrence analysis (Kincaid et al., 2015)

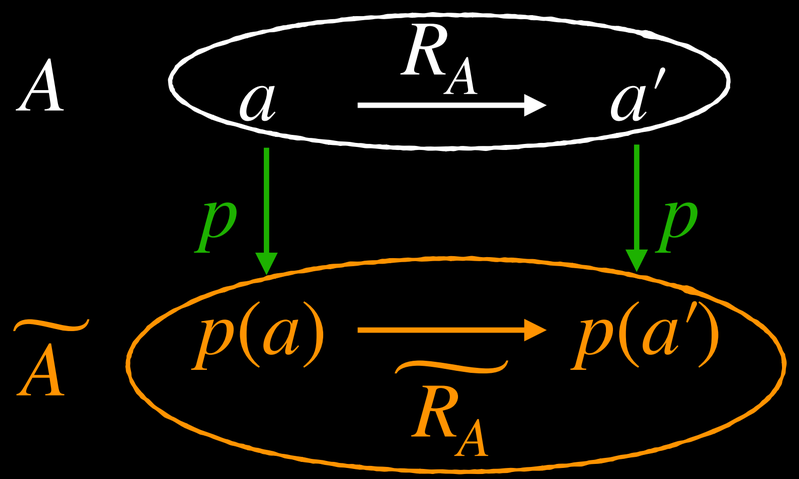
Reduce using **theory of best abstractions** in a monotone manner



Terminating preconditions for linear transition systems with rational eigenvalues based on (Tiwari, 2004)

Simulation as abstraction

- Transition system $A = \langle S_A, R_A \rangle$
- Transition system $\widetilde{A} = \langle \widetilde{S_A}, \widetilde{R_A} \rangle$
- $p : S_A \to \widetilde{S_A}$ is a simulation from transition system A to transition system $\widetilde{A}$ if for all A -transitions $(a, a') \in R_A$, $(p(a), p(a')) \in \widetilde{R_A}$
- Existence of simulation p from A to $\widetilde{A}$ has the consequence that termination of $\widetilde{A}$ implies termination of A
  - If have a terminating condition C for $\widetilde{A}$ then we can get a sufficient terminating condition for A as $p^{-1}(C)$



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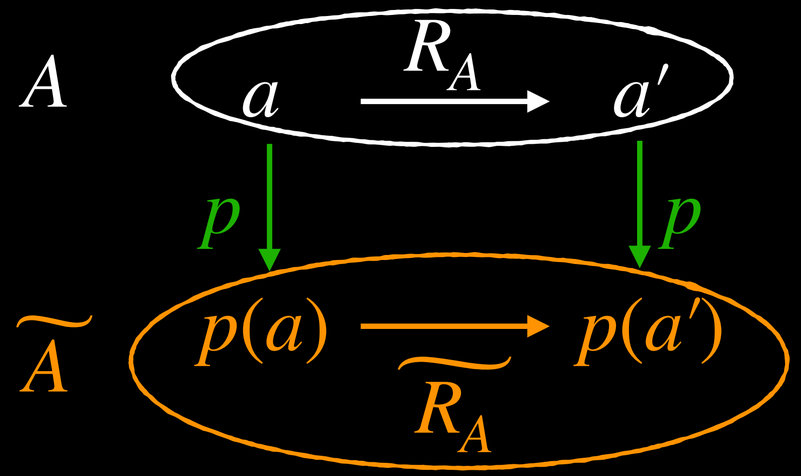
  - If have a terminating condition C for $\widetilde{A}$ then we can get a sufficient terminating condition for A as $p^{-1}(C)$

```
while (x + y ≥ 0) {
    if (__VERIFIER_nondet_int()) {
        x = x - 1;
    } else {
        y = y - 1;
    }
}
```

Transition formula of A:
$x + y \geq 0 \wedge ((x' = x - 1 \wedge y' = y) \vee (y' = y - 1 \wedge x' = x))$

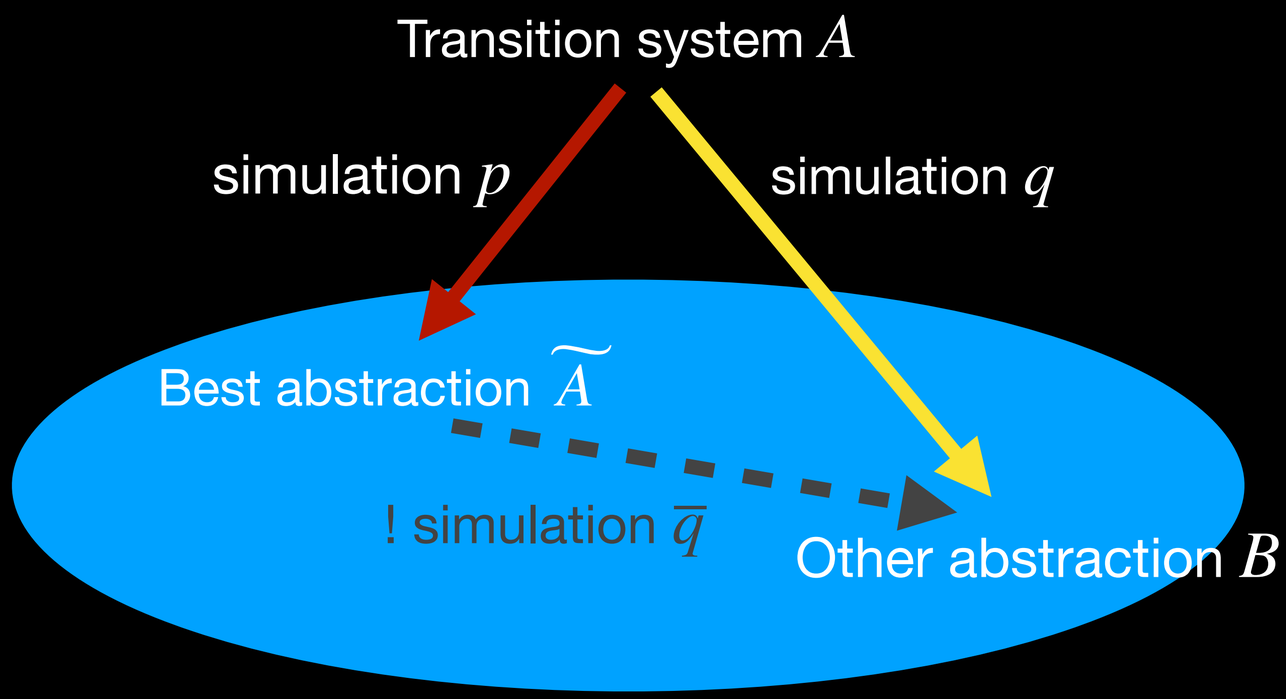
Simulation $s : \begin{bmatrix} x \\ y \end{bmatrix} \to x + y$

Transition formula of $\widetilde{A}$:
$t \geq 0 \wedge t' = t - 1$



Theory of best abstractions

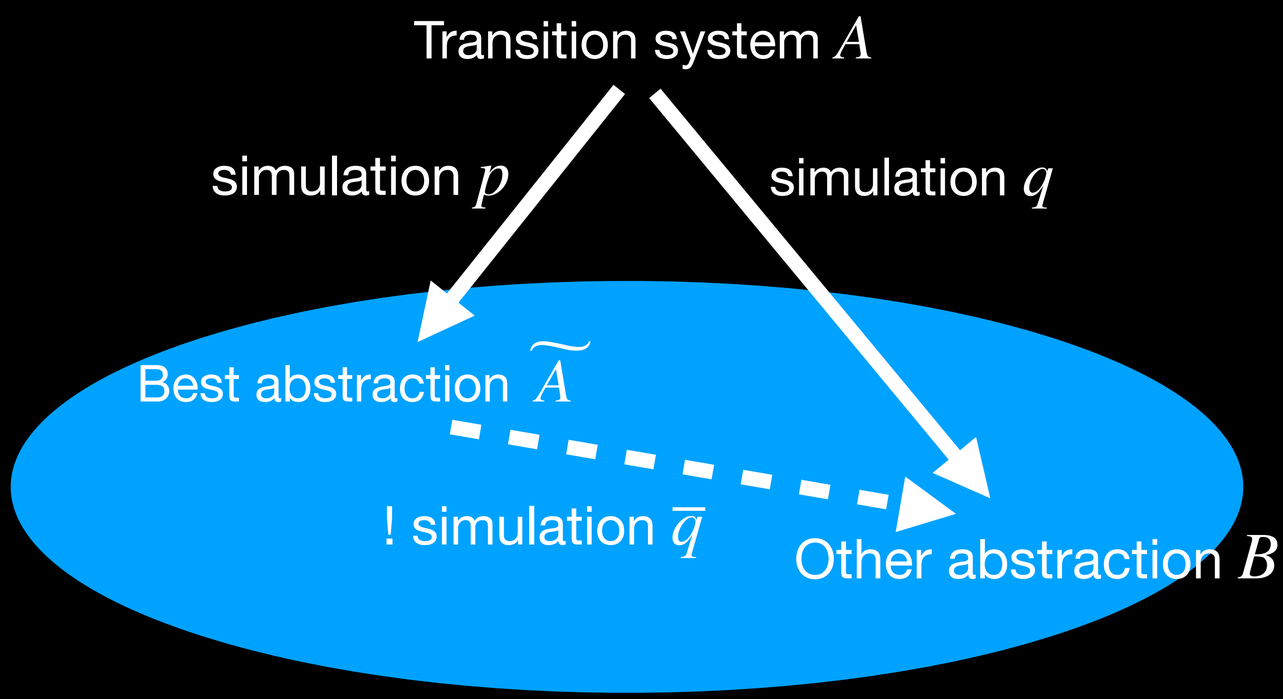
- $\widetilde{A}$ is a best abstraction of A within class $\mathscr{C}$ with respect to a certain class of simulations, e.g., linear simulations



Class $\mathscr{C}$ (over-approximating transition systems)

Best abstraction and monotonicity

- We want to get sufficient terminating conditions for transition system A
- Suppose we have a procedure that computes swf for transition systems in a class $\mathscr{C}$
  - Using the best abstraction $\widetilde{A}$ and simulation $p : A \to \widetilde{A}$, we get sufficient terminating conditions $p^{-1}(\text{swf}(\widetilde{A}))$
  - Using some other abstraction B and simulation $q : A \to B$, we get sufficient terminating conditions $q^{-1}(\text{swf}(B))$
- Under reasonable conditions, using the best abstraction yields the weakest terminating preconditions, compared to using any other abstraction in $\mathscr{C}$

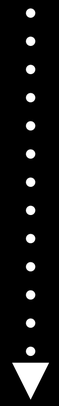


Class $\mathscr{C}$ (over-approximating transition systems)

Termination analysis of the whole program

Compositional Recurrence analysis (Kincaid et al., 2015)

Define swf() operator that finds terminating pre-conditions for a transition formula



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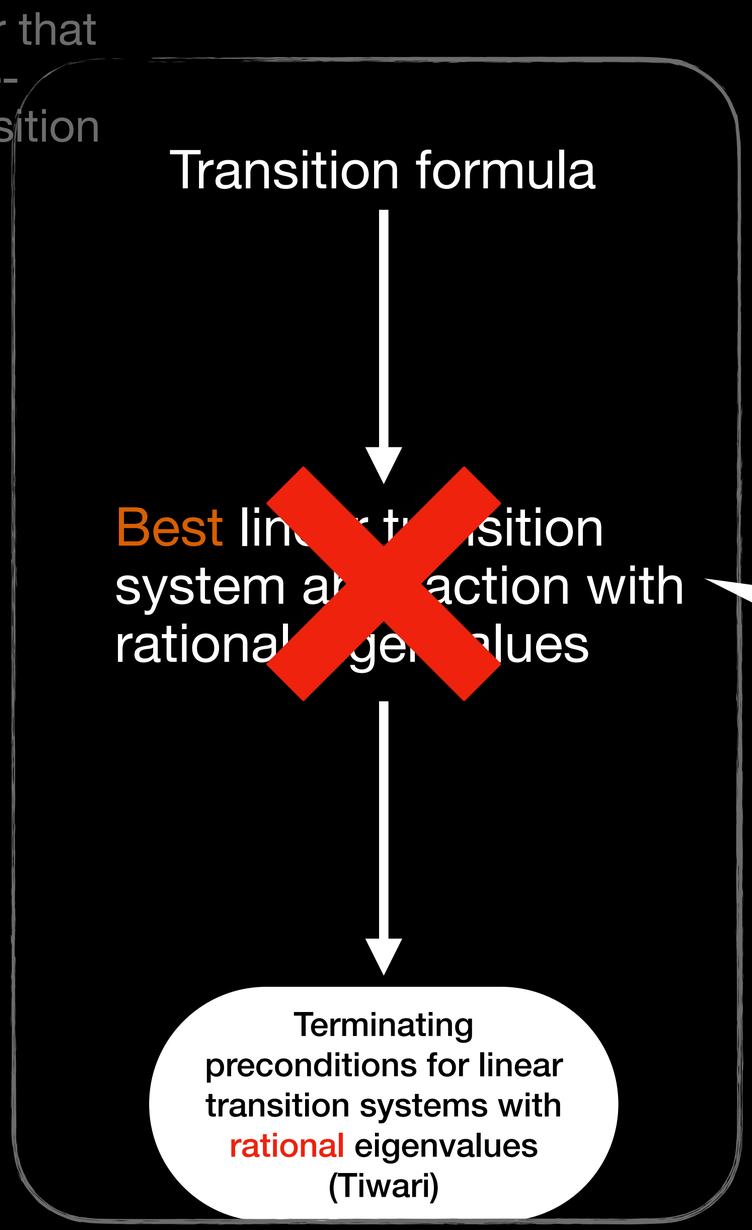
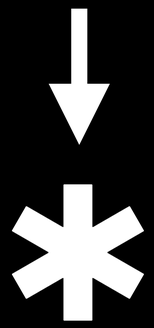




Compositional Recurrence analysis (Kincaid et al., 2015)

Example: Transition formula $x' = x \wedge x' = 0$ has abstractions $x' = x$, $x' = 2x$, $x' = 3x$, etc. but does not have a best abstraction

Theorem: transition formulas in LIA have best abstractions in DATS w.r.t. linear simulations

- DATS: deterministic affine transition system
  - State space $\mathbb{Q}^n$
  - $(\mathbf{u}, \mathbf{v}) \in R \Leftrightarrow \mathbf{v} = T\mathbf{u} + \mathbf{c} \wedge D\mathbf{u} = d$
- cf. TDATS (total deterministic affine transition system)
  - $(\mathbf{u}, \mathbf{v}) \in R \Leftrightarrow \mathbf{v} = P\mathbf{u} + \mathbf{c}$
- Note: transition formulas in LIA does ***not*** necessarily have best abstractions in TDATS w.r.t. linear simulations

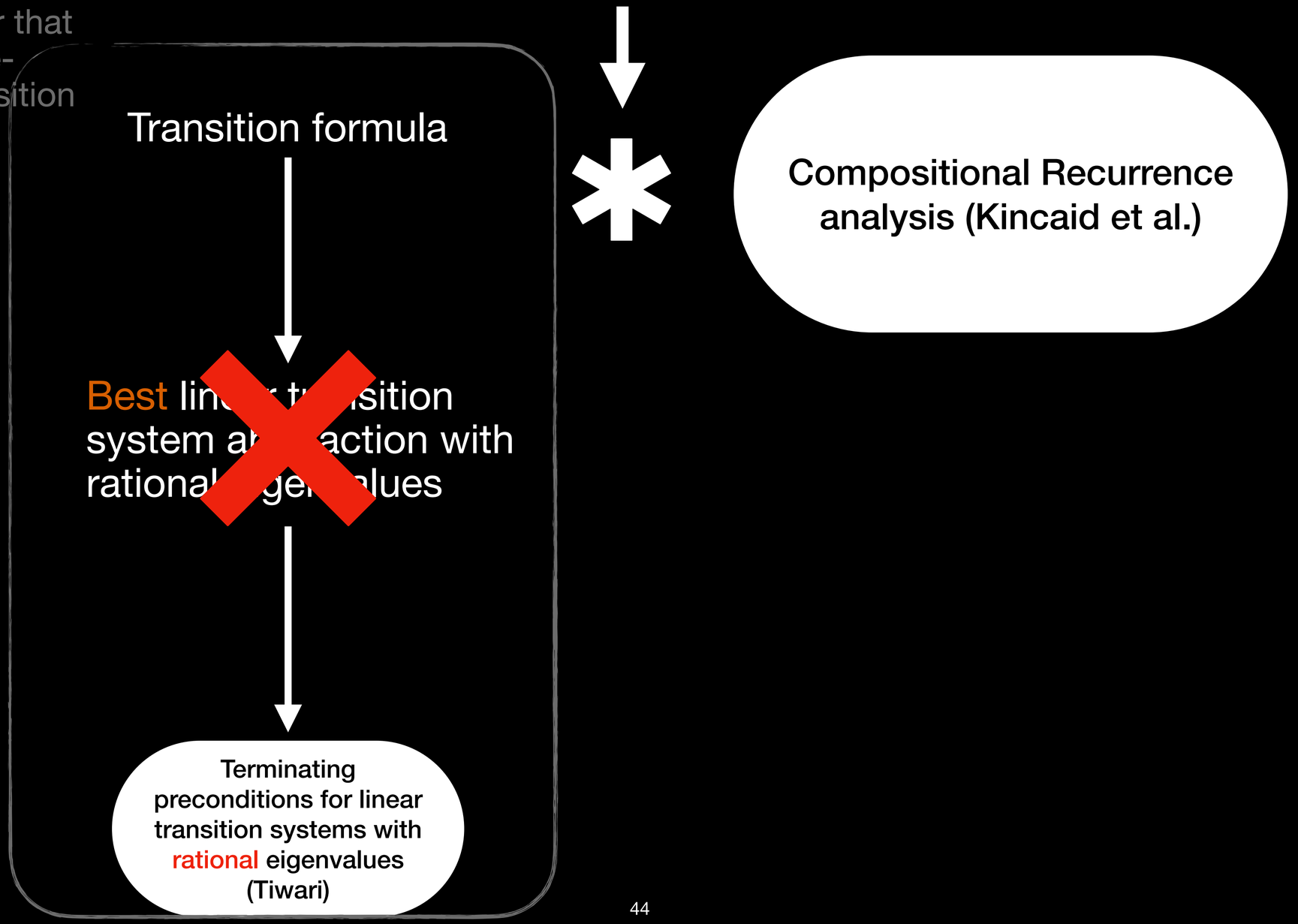
Simplifying DATS

- DATS: deterministic affine transition system
  - $(\mathbf{u}, \mathbf{v}) \in R \Leftrightarrow \mathbf{v} = T\mathbf{u} + \mathbf{c} \wedge D\mathbf{u} = d$
- Simplification 1: easy to homogenize this into a DLTS (deterministic linear transition system)
  - $(\mathbf{u}', \mathbf{v}') \in R \Leftrightarrow \mathbf{v}' = T'\mathbf{u}' \wedge D'\mathbf{u}' = 0$
- Simplification 2: construct a *representation matrix* T_0 that behaves exactly the same as T on **the domain we care about** and maps everything else to 0, then study the exponentiated version of T_0 for the termination of DLTS

- Define the *domain* of a transition relation as $\mathrm{dom}(R) \triangleq \{x \in S : \exists y \,.\, xRy\}$
- Define the *invariant domain* of transition relation R as $\mathrm{dom}^*(R) \triangleq \bigcap_{n \in \mathbb{N}} \mathrm{dom}(R^n)$
  - Intuition: initial states outside of dom^* are certainly terminating pre-states, so only cares about states inside the invariant domain

Termination analysis of the whole program

Define swf() operator that finds terminating pre-conditions for a transition formula



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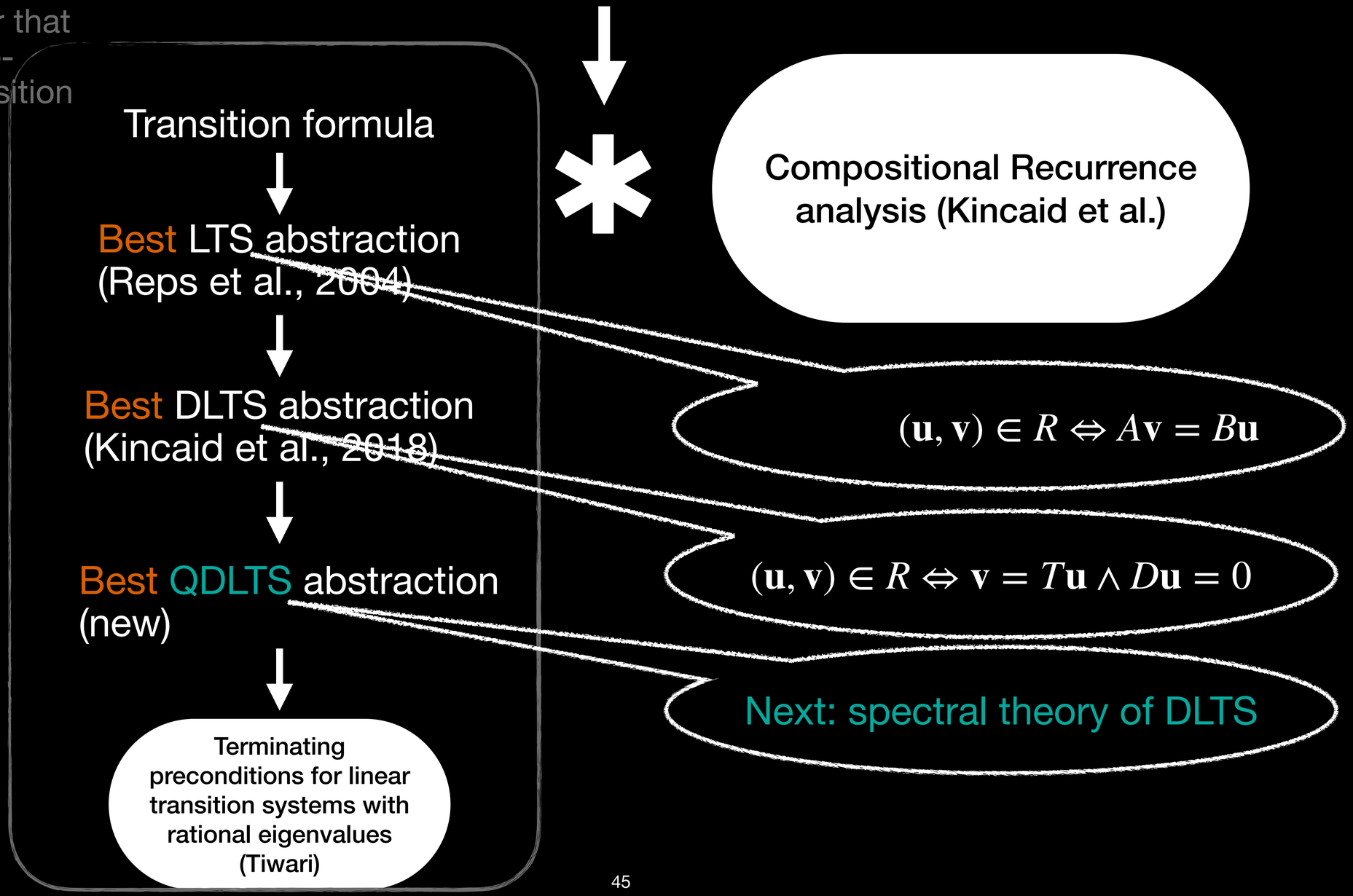
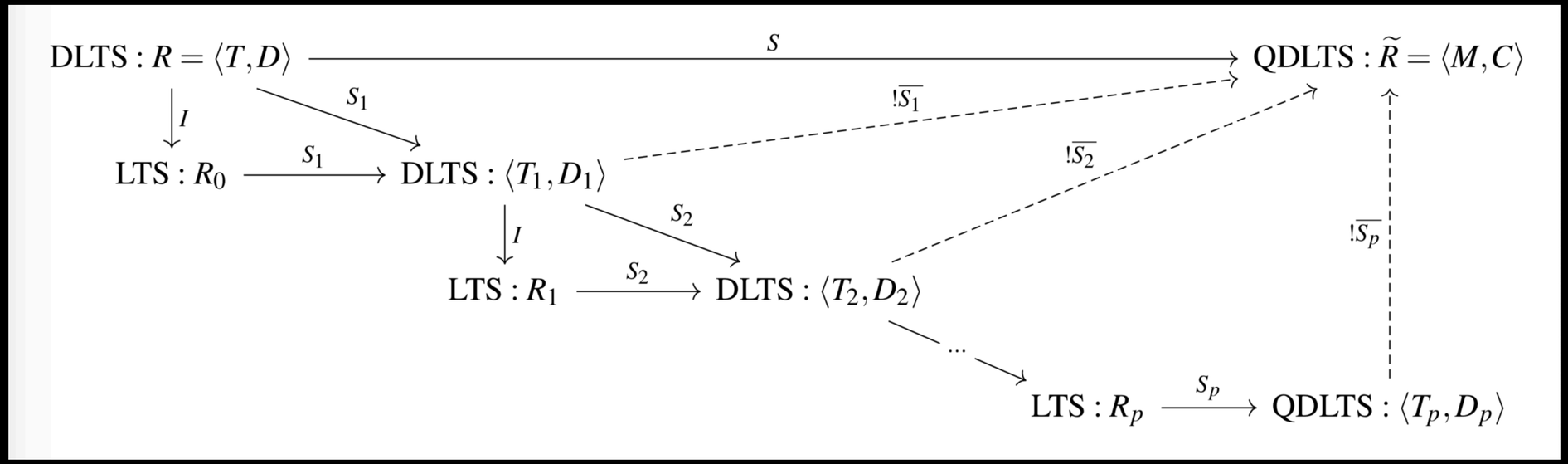


Spectral theory of DLTS

- Spectrum of total deterministic transition systems (TDLTS) $v = Tu$
  - $\mathrm{spec}(T) \triangleq \{\lambda \in \mathbb{C} : \exists v, v \neq 0 . Tv = \lambda v\}$
- Spectrum of deterministic transition systems (DLTS) $v = Tu \wedge Du = 0$
  - $\mathrm{spec}(T, D) \triangleq \{\lambda \in \mathbb{C} : \exists v \in \mathrm{dom}^*(T, D), v \neq 0 . Tv = \lambda v\}$
- QDLTS: DLTS with rational spectrum
- Theorem: the representation T_0 for a QDLTS $v = Tu \wedge Du = 0$ has rational eigenvalues

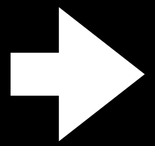
Best QDLTS abstraction

- Theorem: any DLTS has a *best abstraction* as a QDLTS w.r.t. linear simulations
- Proof is constructive and we give an algorithm to compute the best QDLTS abstraction
- Algorithm idea: repeatedly construct DLTS with lower dimensions but remains an over-approximation of the original DLTS, until we obtain a QDLTS or we run out of dimensions



Best QDLTS abstraction restricted to invariant domain is what Tiwari needs

- Properties of a QDLTS:
  - Every iteration is a linear update restricted to a linear space $\mathbf{x}' = T\mathbf{x} \wedge D\mathbf{x} = 0$
  - Spectrum of the above DLTS is all rational



- Another look:
  - Starting from $\mathbf{x}$ within the invariant domain, the updates are characterized by $\mathbf{x}' = T_0\mathbf{x}$
  - Eigenvalues of T_0 are all rational
- initial states outside of the invariant domain are certainly terminating pre-states

Best QDLTS abstraction restricted to invariant domain is what Tiwari needs

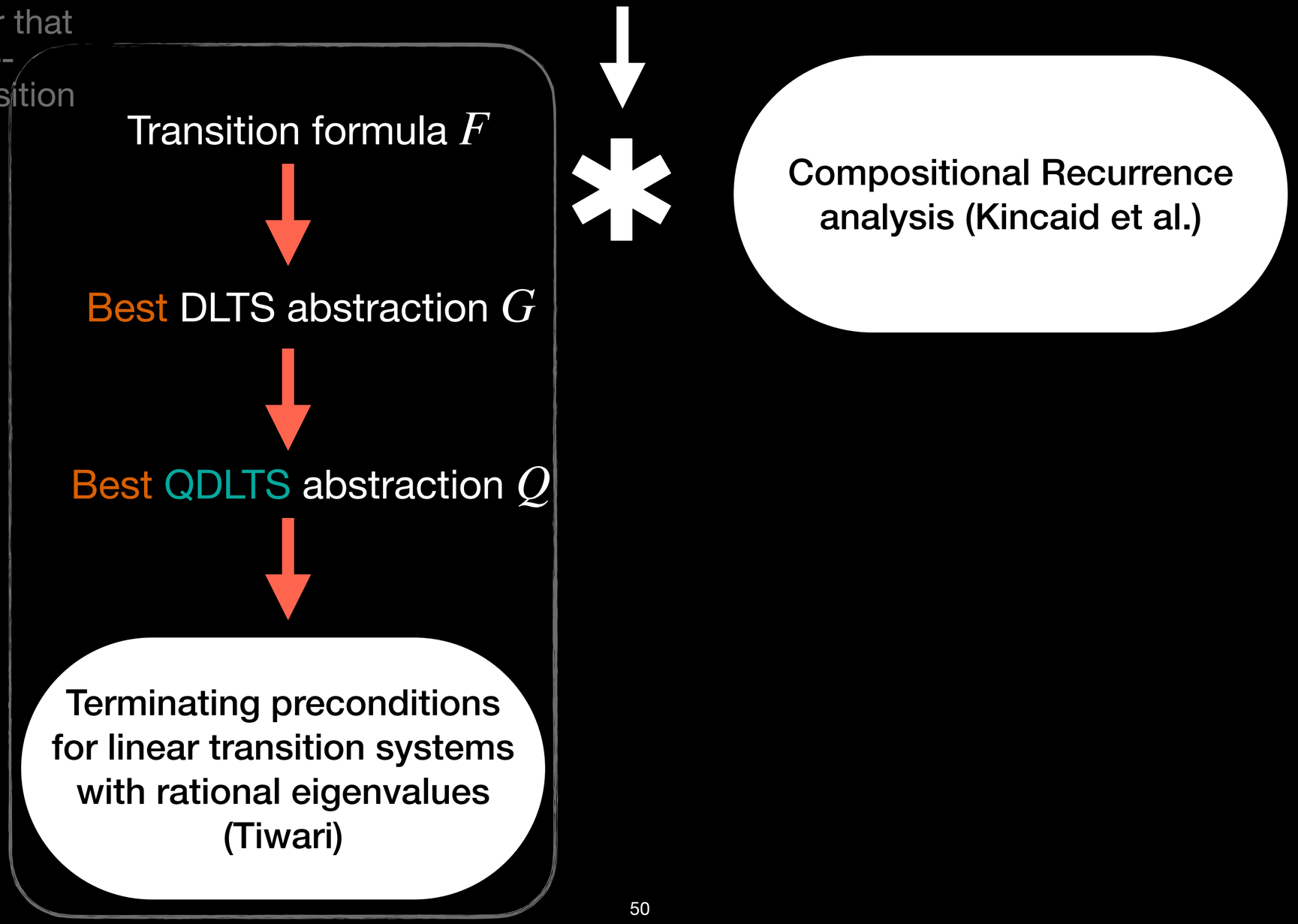
- Tiwari needs:
  - Polyhedral guards
  - Linear updates in the loop body $\mathbf{x}' = A\mathbf{x}$
  - Update matrix A has rational eigenvalues

Procedure to compute the best polyhedral guards given a transition formula and simulation

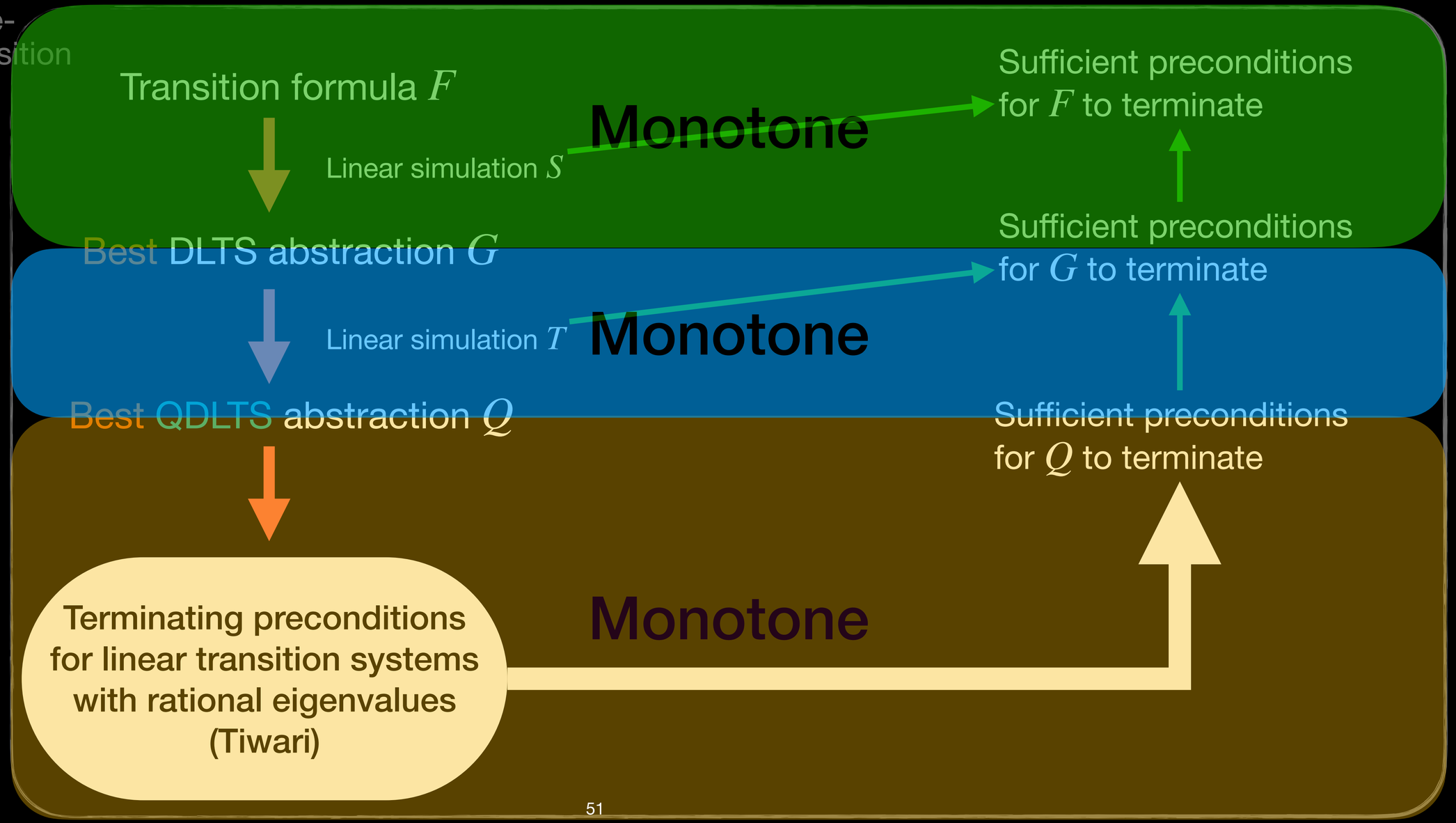
- Another look:
  - Starting from $\mathbf{x}$ within the invariant domain, the updates are characterized by $\mathbf{x}' = T_0\mathbf{x}$
  - Eigenvalues of T_0 are all rational
- initial states outside of the invariant domain are certainly terminating pre-states

Termination analysis of the whole program

Define swf() operator that finds terminating pre-conditions for a transition formula



Define swf() operator that finds terminating pre-conditions for a transition formula

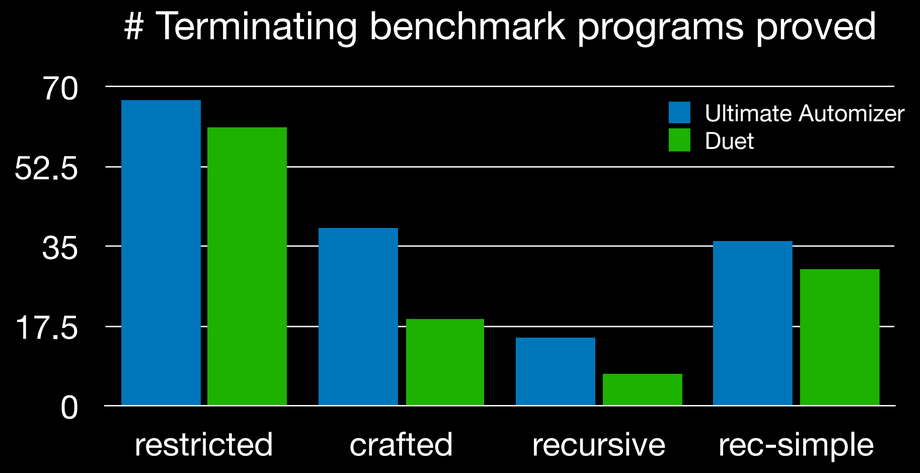


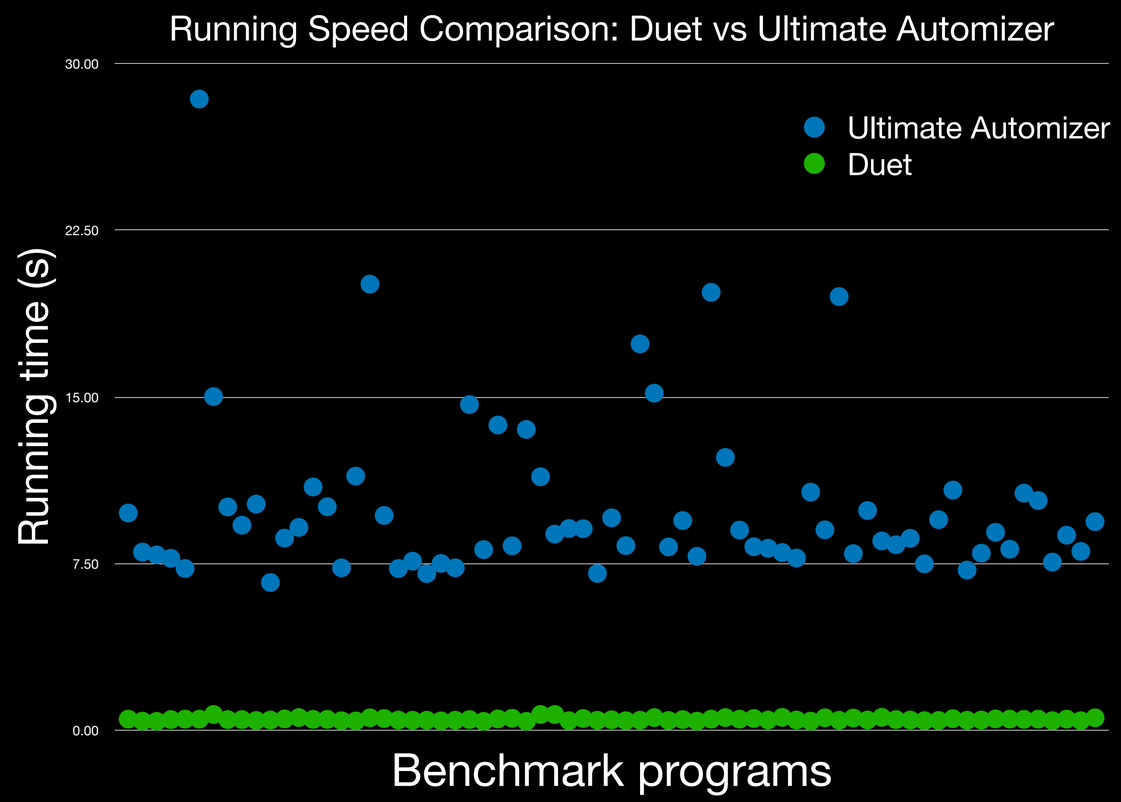
How we achieve compositionality and predictability?

- **Compositionality: compute termination conditions for a program by analyzing its components and composing the results**
  - Extend Tarjan's framework to termination analysis ✓
- **Predictability: give a monotone procedure that computes a sufficient terminating precondition for a program**
  - Exploit the compositional framework
    - Reduce termination of programs to termination of single loops (swf operator)
    - Access to a summary for the body of each loop (atc operator) ✓
  - Utilize decision procedure for linear loops to generate terminating pre-conditions for summaries
    - Abstract transition formulas into linear models that can be handled by Tiwari's method ✓

Preliminary Experimental Results

- SV-COMP termination benchmarks
  - Not geared towards conditional termination
  - Subset containing only terminating programs
- Conclusions
  - Monotonicity and compositionally come at a cost





Future work

- Create more conditional termination benchmarks and evaluate the tool on those benchmarks
- Use termination arguments to enhance loop invariant generation
- Monotone conditional termination analysis for other ranking function templates
- Explore how mathematical theory of dynamical systems could further help with program analysis

References

- [1] B. Cook, A. Podelski, and A. Rybalchenko, “Abstraction Refinement for Termination.,” SAS, 2005.
- [2] A. Tiwari, “Termination of Linear Programs.,” CAV, 2004.
- [3] C. Calcagno, D. Distefano, P. W. O'Hearn, and H. Yang, “Compositional shape analysis by means of bi-abduction.,” POPL, 2009.
- [4] R. E. Tarjan, “A Unified Approach to Path Problems,” Journal of the ACM (JACM), vol. 28, no. 3, pp. 577–593, Jul. 1981.
- [5] T. W. Reps, S. Sagiv, and G. Yorsh, “Symbolic Implementation of the Best Transformer.,” VMCAI, 2004.
- [6] R. Jhala and R. Majumdar, “Software model checking.,” ACM Comput. Surv., 2009.
- [7] L. Gonnord, D. Monniaux, and G. Radanne, “Synthesis of Ranking Functions using Extremal Counterexamples,” {ACM} SIGPLAN Notices, vol. 50, no. 6, pp. 608–618, Jun. 2015.
- [8] T. W. Reps, S. Horwitz, and S. Sagiv, “Precise Interprocedural Dataflow Analysis via Graph Reachability.,” POPL, 1995.
- [9] A. Farzan and Z. Kincaid, “Compositional Recurrence Analysis.,” FMCAD, 2015.
- [10] K. L. McMillan, “Lazy Abstraction with Interpolants.,” CAV, 2006.
- [11] M. Bozga, R. Iosif, and F. Konecný, “Deciding Conditional Termination.,” TACAS, 2012.

Thanks for your attention!